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Recent Progress on Perturbative QCD Fragmentation Functions

Kingman Cheung*
Center for Particle Physics, University of Texas at Austin, Austin TX 78712 U.S.A.
email: cheung@utpapa.ph.utexas.edu

Abstract

The recent development of perturbative QCD (PQCD) fragmentation functions has strong impact on quarkonium production. I shall summarize B_c meson production based on these PQCD fragmentation functions, as well as, the highlights of some recent activities on applying these PQCD fragmentation functions to explain anomalous J/ψ and ψ' production at the Tevatron. Finally, I discuss a fragmentation model based on the PQCD fragmentation functions for heavy quarks fragmenting into heavy-light mesons.

I. Introduction

One of the biggest ever discrepancies between theoretical predictions and experimental data was J/ψ and ψ' production observed by CDF [1] at the Tevatron. With the recent development of perturbative QCD (PQCD) fragmentation functions the experimental data can be accommodated within reasonable uncertainties. In the following I shall briefly describe the essence of PQCD fragmentation functions [2,3,4], then summarize B_c meson production based on these PQCD fragmentation functions [5,6,7] and the highlights of some recent activities on applying the PQCD fragmentation functions to explain anomalous J/ψ and ψ' production [8,9,10,11,12,13,14,15]. Finally, I shall discuss a fragmentation model [16,17,18] based on these PQCD fragmentation functions for the fragmentation of heavy quarks into heavy-light mesons, *e.g.*, $c \to D, D^*$ and $\bar{b} \to B, B^*$. This model is more attractive than previous fragmentation models since it is based on PQCD and the PQCD fragmentation functions have the correct heavy quark behavior.

In general, fragmentation of quarks and gluons lies in the nonperturbative regime so that the fragmentation functions cannot be calculated from first principle. But there is a particular class of fragmentation functions, namely those for heavy quarks or gluons fragmenting into heavy-heavy bound-states, that is calculable in PQCD. Heavy-heavy bound-states refer to heavy quarkonia, $(c\bar{c})$, $(b\bar{b})$, and $(\bar{b}c)$ mesons. To visualize let us consider the hadronization of a heavy quark Q into a meson $Q\bar{q}$, which is schematically shown in Fig. 1. It is the lowest order 1-gluon exchange diagram that describes the hadronization process. As shown in Fig. 1, the central part is the creation of the quark-pair $q\bar{q}$ out of the vacuum, followed by the binding of $\bar{q}$ to Q to form the meson $Q\bar{q}$. Therefore, the natural scale of this process is of order m_q, specifically we choose it to be $2m_q$. Figure 1 could be used to picture the

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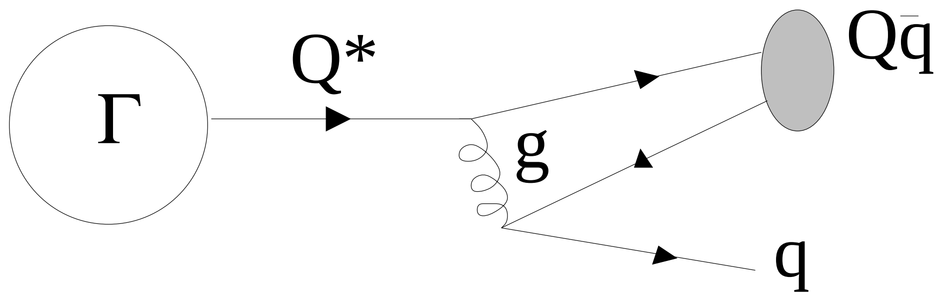

Figure 1: Schematical diagram for a heavy quark Q fragmenting into $(Q\bar{q})$ and q.

fragmentation of a heavy quark into a heavy-light meson when $Q = b, c$ and $q = u, d$. The natural scale is then equal to $2m_{u,d}$, which is close to $\Lambda_{\rm QCD}$, so that we do not expect the process to be calculable by PQCD. The nonperturbative physics involved demands a model calculation.

On the other hand, figure 1 can also be used to describe the production of $(b\bar{b})$, $(c\bar{c})$, and $(\bar{b}c)$ mesons when $Q = b, c$ and $q = b, c$. But the main difference is that the natural scale of the process is now of order $2m_{b,c}$, which is much larger than $\Lambda_{\rm QCD}$. In other words, the fragmentation process can be reliably calculated as an expansion in $\alpha_s(2m_{b,c})$. While the diagram in Fig. 1 represents the lowest order α_s^2 term; higher order corrections can be systematically calculated. The arguments for the calculability of gluon fragmentation into charmonium and bottomium within PQCD are essentially the same.

Even though we can calculate the fragmentation process by PQCD, there are bound-state effects that have to be taken care of. The nonperturbative bound-state effects can be parameterized by, *e.g.*, wavefunctions or derivatives of wavefunctions of the bound-states at the origin. For the case of S-wave mesons there is only one nonperturbative parameter – $R(0)$, the wavefunction at the origin; while for P-wave mesons there are two nonperturbative parameters, which correspond to the color-singlet and color-octet mechanisms. Fragmentation into different spin-orbital states can be obtained by using the appropriate spin projections.

The b, c quark fragmentation functions can be calculated by this expression

$$D_{Q\to Q\bar{q}}(z) \;=\; \frac{1}{16\pi^2}\int ds\,\theta\left(s-\frac{M^2}{z}-\frac{m_q^2}{1-z}\right)\lim_{p_{Q_0}/m_Q\to\infty}\frac{|\mathcal{M}|^2}{|\mathcal{M}_0|^2} \qquad (1)$$

where $Q, q = b, c$, $M = m_Q + m_q$ is the mass of the meson, $\mathcal{M}$ is the amplitude for producing a $Q\bar{q}$ and q from an off-shell Q^* with virtuality $s = p_Q^2$ (Fig. 1), p_Q is the 4-momentum of the heavy quark Q, and $\mathcal{M}_0$ is the amplitude for producing the heavy quark Q with the same 3-momentum $\vec{p}_Q$. Here I only present the results for $\bar{b}\to(\bar{b}c)$ in the S-wave states [4]:

$$\begin{aligned} D_{\bar{b}\to\bar{b}c(^1S_0)}(z,\mu_0) \;&=\; N\,\frac{rz(1-z)^2}{(1-(1-r)z)^6}\Big[6-18(1-2r)z+(21-74r+68r^2)z^2 \\ &\quad -2(1-r)(6-19r+18r^2)z^3+3(1-r)^2(1-2r+2r^2)z^4\Big] \qquad (2)\end{aligned}$$

for the ground state denoted by B_c, and

$$D_{\bar b\to\bar c(^3S_1)}(z,\mu_0) = 3N\frac{rz(1-z)^2}{(1-(1-r)z)^6}\Big[2-2(3-2r)z+3(3-2r+4r^2)z^2 -2(1-r)(4-r+2r^2)z^3+(1-r)^2(3-2r+2r^2)z^4\Big] \quad (3)$$

for the first excited state denoted by B_c^*, where $N = 2\alpha_s(2m_c)^2|R(0)|^2/(81\pi m_c^3)$ and $r = m_c/(m_b+m_c)$. The results for longitudinally and transversely polarized B_c^* mesons can be found in Ref. [16], and the results for P-wave can be found in Refs. [19,20]. The fragmentation functions for charm quark into charmonium can be obtained by putting $r = 1/2$ in the above expressions, and can be found in Ref. [3,19,21]. Gluon fragmentation functions into charmonium and bottomium can be calculated in a similar fashion, and the results are in Ref. [2] for S-wave, Ref. [22] for P-wave, and Ref. [23] for $^1D_2$ (see also Refs. [24] for a derivation from a field theoretical definition). Likewise, the photon and lepton fragmentation functions into charmonium were calculated in Ref. [25].

A couple of remarks about these PQCD fragmentation functions are in order. (i) The scale of the PQCD fragmentation functions calculated should be of order of the heavy quark mass m_Q. We choose it to be m_Q+2m_q, which is the minimum virtuality of the fragmenting quark. In the case of gluon fragmentation functions, the scale is set at $3m_Q$. These fragmentation functions obey the usual Altarelli-Parisi evolution equations such that fragmentation functions at higher scale can be obtained by evolving the Altarelli-Parisi equations. (ii) The inputs to these PQCD fragmentation functions are simply the masses of charm and bottom quarks, and the wavefunctions at the origin (also derivatives of the wavefunctions and color-octet wavefunctions for higher orbital states). All of these quantities can be reliably obtained from potential models or from lattice simulations. Since the inputs can be reliably obtained from other sources, these PQCD fragmentation functions have high predictive power.

In Sec. II the fragmentation approach for calculating the B_c meson production is described, and the same approach has been used to calculate the fragmentation contribution to the ψ and ψ' production, as will be summarized in Sec. III. Sec. IV describes a fragmentation model.

II. B_c Meson Production

The fragmentation approach has been used in calculating the fragmentation contribution to the production of $\bar bc$, charmonium, and bottomium. This approach is based on factorization, in which the production process is separated into the production of high energy partons (quarks and gluons) and the fragmentation of these partons into the meson. In this section, I summarize the results from a series of studies [5,6,7] on the production rates of the S-wave and P-wave $(\bar bc)$ mesons, as well as, the inclusive production of B_c. $(\bar bc)$ mesons belong to another heavy-quark bound state family, which is made up of a $\bar b$ antiquark and a c quark. The spectroscopy for the spin-orbital states is similar to that of charmonium and bottomium, and $(\bar bc)$ can be obtained by interpolating between charmonium and bottomium. According to potential models [26], the 1S, 1P, 1D, 2S, and possibly the whole set of 2P states lie below the BD threshold. A very peculiar feature of the excited $(\bar bc)$ states is that the

annihilation channel is suppressed relative to the electromagnetic or hadronic transitions into lower-lying states. Therefore, when an excited state is produced it will cascade into the ground state with emission of photons and pions. Hence, all the 1S, 1P, 1D, 2S, and probably 2P states contribute to the inclusive production of the ground state B_c meson. The D-wave fragmentation functions are not available yet but they are expected to contribute only a very small fraction.

The differential cross section for producing a $(\bar{b}c)$ meson in a spin-orbital state H is given by

$$\begin{aligned}\frac{d\sigma}{dp_T}(p\bar{p} \to H(p_T)X) &= \sum_{ij} \int dx_1 dx_2 dz f_{i/p}(x_1,\mu) f_{j/\bar{p}}(x_2,\mu) \left[\frac{d\hat{\sigma}}{dp_T}(ij \to \bar{b}(p_T/z)X,\, \mu) \right. \\ & \left. \times D_{\bar{b}\to H}(z,\mu) + \frac{d\hat{\sigma}}{dp_T}(ij \to g(p_T/z)X,\mu)\; D_{g\to H}(z,\mu) \right] , \qquad (4)\end{aligned}$$

where $f_{i/p}(x)$'s are the parton distribution functions, $d\hat{\sigma}$'s are the subprocess cross sections, and $D_{i\to H}(z,\mu)$'s represent the parton fragmentation functions at the scale μ. The factorization scale μ is chosen in the order of the p_T of the parton, so as to avoid large logarithms in $d\hat{\sigma}$, and the resulting large logarithms of order μ/m_b in $D_{i\to H}(z,\mu)$ can be summed by the Altarelli-Parisi equations. The gluon fragmentation functions at the initial scale are α_s suppressed relative to the $\bar{b}$ fragmentation functions, so we simply take the initial gluon fragmentation functions to be zero. This is justified since the majority of the gluon fragmentation comes from the Altarelli-Parisi evolution, and we called it the induced gluon fragmentation functions [6]. The $\bar{b}$ and gluon fragmentation functions into a $(\bar{b}c)$ state H satisfy the following evolution equations

$$\mu\frac{\partial}{\partial\mu} D_{\bar{b}\to H}(z,\mu) = \int_z^1 \frac{dy}{y} P_{\bar{b}\to\bar{b}}(z/y,\mu)\; D_{\bar{b}\to H}(y,\mu) + \int_z^1 \frac{dy}{y} P_{\bar{b}\to g}(z/y,\mu)\; D_{g\to H}(y,\mu)\,, \quad (5)$$

$$\mu\frac{\partial}{\partial\mu} D_{g\to H}(z,\mu) = \int_z^1 \frac{dy}{y} P_{g\to\bar{b}}(z/y,\mu)\; D_{\bar{b}\to H}(y,\mu) + \int_z^1 \frac{dy}{y} P_{g\to g}(z/y,\mu)\; D_{g\to H}(y,\mu)\,, \quad (6)$$

where $P_{i\to j}$ are the usual Altarelli-Parisi splitting functions. The initial conditions to the above equations are simply the initial $\bar{b}$ fragmentation functions and the initial gluon fragmentation functions, which are set to zero. We can also examine the relative importance of these fragmentation functions. The initial $D_{\bar{b}\to H}(z,\mu_0)$ is of order α_s^2, even when it is evolved to a higher scale it is still of order α_s^2. In contrast, the initial $D_{g\to H}(z,\mu_0)$ is of order α_s^3, but at a higher scale μ it is of order $\alpha_s^3 \ln(\mu/\mu_0)$ via the Altarelli-Parisi evolution. Therefore, at a sufficiently large scale the induced $D_{g\to H}(z,\mu)$ is as important as the $\bar{b}$ quark fragmentation.

The resulting p_T spectra for the S-wave and P-wave states are shown in Fig. 2. We can now predict the inclusive production rate of the ground state B_c meson. We add up the cross sections from all S-wave and P-wave production, and the inclusive cross sections of B_c as a function of $p_T^{\rm min}(B_c)$ is shown in Table I at the Tevatron. Variations with factorization scale between $\mu_R/2$ to $2\mu_R$, where $\mu_R = \sqrt{p_T^2(\text{parton}) + m_b^2}$, are also illustrated. The variation is at worst a factor of two, and substantially reduced at $p_T^{\rm min} > 10$ GeV. With a production cross section of about 5 nb and 100 pb^{-1} integrated luminosity at the Tevatron there are about 5×10^5 B_c^+ mesons. The detection mode for the B_c meson will be $B_c \to J/\psi + X$,

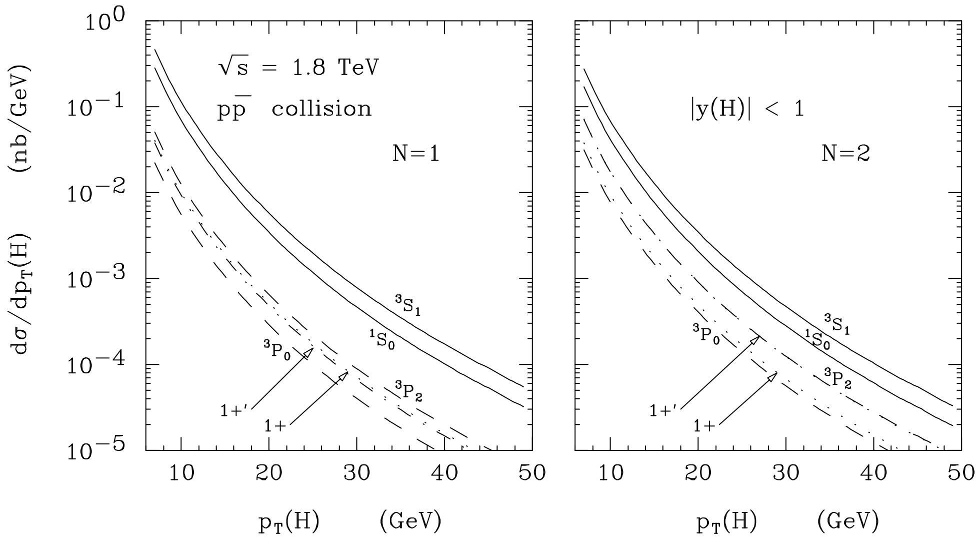

Figure 2: Differential cross sections versus the p_T of the $(\bar{b}c)$ mesons in various spin-orbital states with $N=1$ and $N=2$, respectively. The acceptance cuts are $p_T(H) > 6$ GeV and $|y(H)| < 1$.

where X can be a π^+, ρ^+, or $\ell^+\nu_l$, and J/ψ can be detected easily through its leptonic decay modes. When X is $e^+\nu_e$ or $\mu^+\nu_\mu$, it will be a striking signature of three-charged leptons coming from a common secondary vertex. The combined branching ratio of $B_c \to J/\psi\ell^+\nu_\ell \to \ell'^+\ell'^-\ell^+\nu_\ell$ ($\ell, \ell' = e, \mu$) is about 0.2%. This implies that there will be of order 10^3 such distinct events for 100 pb^{-1} luminosity at the Tevatron. However, this mode does not afford the full reconstruction of the B_c. If X is some hadronic states, *e.g.*, pions, the events can be fully reconstructed and the B_c meson mass can be measured. The process $B_c \to J/\psi + \pi^+ \to \ell^+\ell^-\pi^+$ is likely to be the discovery mode for B_c. Its combined branching ratio is about 0.03%, which implies about 300 such distinct events at the Tevatron with a luminosity of 100 pb^{-1}. The production rates of B_c at the LHC will be of order 10^9 with 100 fb^{-1} luminosities, promising a very exciting experimental program on B_c mesons at the LHC.

There also exist complete $\mathcal{O}(\alpha_s^4)$ calculations on the production of S-wave B_c mesons [27,28]. There is a controversy whether the fragmentation contribution will dominate the

Table I: The inclusive production cross sections for the B_c meson at the Tevatron including the contributions from all the S-wave and P-wave states below the BD threshold as a function of $p_T^{\min}(B_c)$. The acceptance cuts are $p_T(B_c) > 6$ GeV and $|y(B_c)| < 1$.

$p_T^{\min}$ (GeV)	σ (nb)		
	$\mu = \frac{1}{2}\mu_R$	$\mu = \mu_R$	$\mu = 2\mu_R$
6	2.81	5.43	6.93
10	0.87	1.16	1.22
15	0.26	0.29	0.26
20	0.098	0.097	0.083

production of B_c mesons at the large p_T regions over the non-fragmentation (recombination) contribution. The controversy arises because the set of fragmentation diagrams is a gauge-invariant subset of the whole set of Feynman diagrams at the order α_s^4. So there is a competition between the fragmentation diagrams and the recombination diagrams. Nevertheless, the bottom line is that the fragmentation approach identifies the correct scale for the fragmentation diagrams, and should give a lower bound on the production cross section of B_c mesons.

III. J/ψ and ψ' Production

In this section, I highlight some recent activities on J/ψ and ψ' production at the Tevatron. Before the recent CDF data [1], the dominant mechanism of J/ψ production at high p_T region was believed to be the fusion mechanism, $gg \to \chi_{cJ} g$, followed by the radiative decay of $\chi_{cJ} \to J/\psi + \gamma$, while for ψ' production the dominant mechanism is $gg \to \psi' g$ because of the absence of $\chi_{cJ}(2P)$ states. But the CDF measurements on J/ψ and ψ' exposed large discrepancies between theoretical predictions and experimental data. The discrepancies demonstrate that there must be either some other unknown production mechanisms or simply that perturbative QCD is not valid in this case.

In order that perturbative QCD is still the means to understand the production of heavy quarkonia, it is advantageous to consider higher order contributions, which are more important than the lowest order fusion mechanism. It was shown explicitly in Ref. [9] that the contributions to J/ψ and ψ' production from gluon, charm quark, and photon fragmentation are more important than the lowest order fusion mechanisms beyond certain values of p_T (see Fig. 3.) Among all the fragmentation contributions that are relevant to J/ψ production, the largest one comes from gluon fragmentation into χ_{cJ} followed by the radiative decay $\chi_{cJ} \to J/\psi + \gamma$ [22]. The gluon fragmentation $D_{g\to\chi_{cJ}}(z)$ consists of two pieces, one of which is the color-singlet part of order α_s^2 and the other piece is the color-octet part of order α_s. When the fragmentation contributions are included, the theoretical prediction matches the experimental data within a factor of $2-3$ [8,9,10] (see Fig. 4), which is within the uncertainties from the mass of charm quark, the factorization scale, higher order QCD corrections, and relativistic corrections.

While anomalous J/ψ production seems to be solved, however, the data for ψ' production is still a factor of $20-30$ above the theoretical prediction, even after including the fragmentation contributions (see Fig. 4). The $\chi_{cJ}(2P)$ states are predicted to be above the $D\bar{D}$ threshold and therefore do not contribute to ψ' production. This discrepancy is sometimes referred as the ψ' anomaly. Of course, there have been speculative solutions to the anomaly. The most obvious solution is the hypothesis that $\chi_{cJ}(2P)$ states are metastable such that they decay with appreciable branching ratios into ψ' [11,12,13,14]. According to potential models, the $\chi_{cJ}(2P)$ states are above the $D\bar{D}$ threshold, but the decay of $\chi_{cJ} \to D\bar{D}$ might be suppressed due to a D-wave suppression. Therefore, an appreciable fraction of $\chi_{cJ}(2P)$ can decay into ψ'. In order to explain the ψ' anomaly, a branching ratio $B(\chi_{cJ}(2P) \to \psi' + \gamma) \approx 5-10\%$ is needed. However, such a large branching ratio is unflavored by potential models. There is also another mechanism due to Braaten and Fleming [15], who proposed that the production is via gluon fragmentation into a color-octet ${}^3S_1^{(8)}$

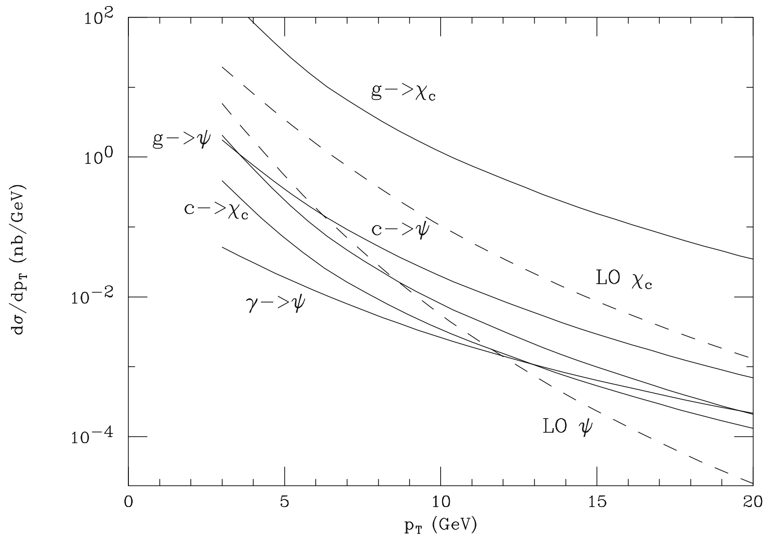

Figure 3: Contributions to the differential cross section for inclusive J/ψ production at the Tevatron: fragmentation into ψ (solid curves), and the leading order contributions (dashed curves). (Taken out from Ref. [9].)

state, which then nonperturbatively emits a pair of gluons to make the transition into ψ' state. This mechanism is suppressed by powers of v, which is the relative velocity between c and $\bar{c}$ inside the charmonium. But on the other hand, this fragmentation is flavored by two powers of α_s compared to the color-singlet fragmentation function (α_s^3). It means that this fragmentation mechansim could be potentially large because the corresponding fragmentation function is only of order α_s, though suppressed by powers of v. The major uncertainty is the determination of the nonperturbative parameter associated with the soft emission of the gluon-pair in color-octet 0^+ state. This mechanism can be tested rather easily because the ψ' produced will be entirely transversely polarized [15,12], and the polarization can be easily measured experimentally by looking at the angular distribution of the muon pair in the decay of ψ'. If this mechanism is tested to be important, it will significantly affect all hadro-, photo-, and electro- production of charmonium. In fact, a very recent analysis indicated that only about one third of the prompt J/ψ comes from χ_{cJ} decays, while the rest is from direct J/ψ production. It means that there is another important production mechanism other than the gluon fragmentation into χ_{cJ}'s followed by the radiative decay of χ_{cJ}'s into J/ψ. This might be the hint showing the importance of Braaten-Fleming's color-octet mechanism [15] in J/ψ production as well.

IV. A Fragmentation Model

In this section I describe a fragmentation model [16,17,18] for the fragmentation of heavy quarks into heavy-light mesons. This model is based on the PQCD fragmentation functions that are presented in Eqs. (2) and (3) for S-wave mesons, and in Ref. [19] for P-wave mesons. But since most of the experimental data are on S-wave states, I concentrate on Eqs. (2)–(3).

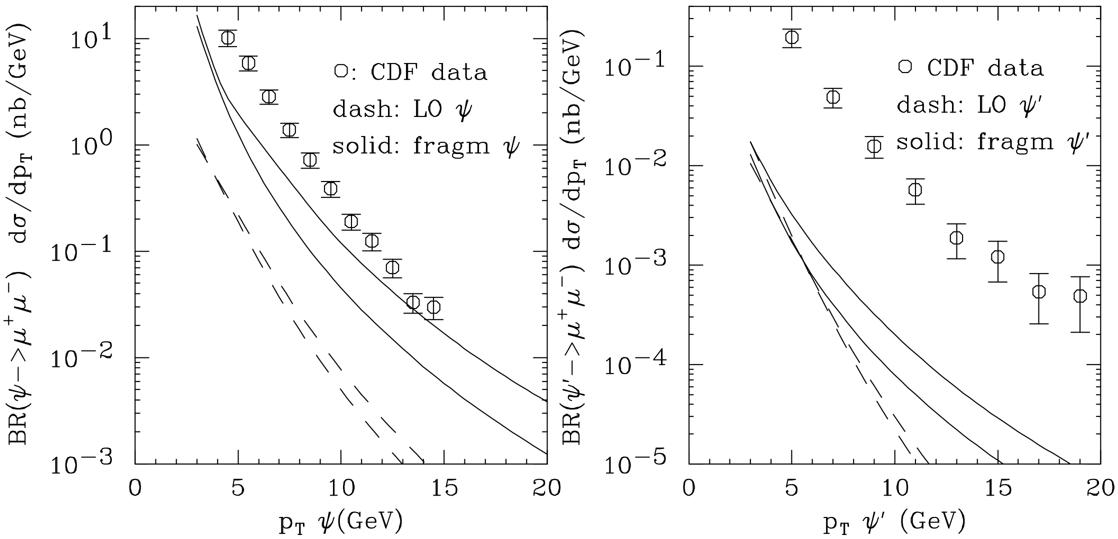

Figure 4: Preliminary CDF data for prompt ψ and ψ' production with theoretical predictions of the total fragmentation contributions (solid) and the leading order fusion contributions (dash).

Let me rewrite the fragmentation functions here:

$$\begin{aligned} D_{Q\to Q\bar{q}(^1S_0)}(z,\mu_0) &= N\,\frac{rz(1-z)^2}{(1-(1-r)z)^6}\Big[6-18(1-2r)z+(21-74r+68r^2)z^2 \\ &\quad -2(1-r)(6-19r+18r^2)z^3+3(1-r)^2(1-2r+2r^2)z^4\Big]\,, \end{aligned} \tag{7}$$

$$\begin{aligned} D_{Q\to Q\bar{q}(^3S_1)}(z,\mu_0) &= 3N\,\frac{rz(1-z)^2}{(1-(1-r)z)^6}\Big[2-2(3-2r)z+3(3-2r+4r^2)z^2 \\ &\quad -2(1-r)(4-r+2r^2)z^3+(1-r)^2(3-2r+2r^2)z^4\Big]\,, \end{aligned} \tag{8}$$

where N, instead of being given in terms of the wavefunction and the running coupling constant, is now treated as a free parameter governing the overall normalization together with another free parameter r, r being the mass ratio $m_q/(m_Q+m_q)$. This model, in the limit $r\to 0$, approaches the fragmentation of a very heavy quark into a heavy-light meson. In reality, the heavy quark is either b or c, and the light component is u, d, or s, and therefore r is a small parameter. In principle, when the lighter constituent quark becomes a light quark, there is that nonperturbative physics involved in the fragmentation process. But we do expect our PQCD fragmentation functions with N and r as free parameters can at least provide a qualitative picture and hence a reasonable model for fragmentation into heavy-light mesons. This model is suitable for $b\to B, B^*, B^{**}$, ... and $c\to D, D^*, D^{**}$, ... mesons.

This fragmentation model has certain advantages over previous models [29] in the literature. First of all, this model lies on a firm basis of PQCD. It is rigorously correct in the limit when $m_q \gg \Lambda_{\rm QCD}$ and higher order corrections can be systematically calculated. The spirit of our model is the continuation of m_q to a value close to $\Lambda_{\rm QCD}$. The most obvious advantage of this model is the ability to predict different results for different spin-orbital

states with only two parameters, in contrast to the Peterson model. Another advantage is that the fragmentation functions for the same orbital angular momentum share the same paramter N, as shown in Eqs (7) – (8). This is a substantial improvement when ratios of the fragmentation functions are measured, in which the N dependence cancels out. For example, our model can predict the ratio $P_V = V/(V+P)$ as a function of r only, where V is the vector meson and P is the pseudoscalar.

Another theoretical issue is that the PQCD fragmentation functions are consistent with heavy quark symmetry, which I explain in more detail next. According to an analysis using the heavy quark effective theory (HQET) [30], the fragmentation function for a heavy quark Q into a hadron H_Q containing a single heavy quark Q at the heavy quark mass scale m_Q is

$$D_{Q\to H_Q}(z) = \frac{1}{r}a(y) + b(y) + \mathcal{O}(r) \tag{9}$$

which is a heavy quark mass expansion in $r = \frac{m_{H_Q}-m_Q}{m_{H_Q}}$ and y is a rescaled variable of z, $y = \frac{1-(1-r)z}{rz}$. The leading term is of order $1/r$, i.e., m_Q, while the next-to-leading term is of order r^0. The PQCD fragmentation functions in Eqs. (7) and (8) can be expanded in powers of r and reexpressed in terms of y, as

$$\begin{aligned} D_{Q\to Q\bar{q}(^1S_0)}(z) &= \tfrac{N(y-1)^2}{y^6}\left(\tfrac{1}{r}(3y^2+4y+8) + (3y^3+15y^2+8y-8) + ...\right) \\ D_{Q\to Q\bar{q}(^3S_1)}(z) &= \tfrac{N(y-1)^2}{y^6}\left(\tfrac{3}{r}(3y^2+4y+8) - 3(y^3+y^2-8y+8) + ...\right) \end{aligned} \tag{10}$$

The above expansions in powers of r are consistent with the HQET analysis [30] that the leading term is order $1/r$. Actually, if examined more carefully, the leading terms of the $^1S_0$ and $^3S_1$ expressions are exactly in the ratio of 1:3, which is the value predicted by heavy quark spin symmetry. The next-to-leading terms are not in the ratio of 1:3, and they explicitly break the spin symmetry. This fact also prompted us to derive the PQCD fragmentation functions independently from the HQET Lagrangian [18]. By using the leading and the next-to-leading terms of the HQET Lagrangian, we succeeded in obtaining the same results as the heavy quark mass expansions in Eqs. (10). Therefore, with the consistency with heavy quark symmetry and HQET we have more confidence in applying our PQCD fragmentation functions as a fragmentation model for heavy quark fragmenting into heavy-light mesons, namely, $\bar{b} \to B, B^*, B^{**}$ and $c \to D, D^*, D^{**}$,... When more data on P-wave mesons are available, comparisons with the P-wave fragmentation functions can also be made. For the moment the data are more or less entirely on the S-wave states. I shall demonstrate a couple of comparisons between the predictions by this model and the experimental data. We shall look at P_V and $\langle z\rangle$.

(A) P_V for the charm system is defined as $P_V = D^*/(D+D^*)$, which is a measure of the population of D^* in a sample of D and D^* mesons. Since fragmentation is the dominant production mechanism for charm mesons and D, D^* mesons dominate, P_V can be expressed in terms of the fragmentation functions

$$P_V = \frac{\int dz D_{c\to D^*}(z)}{\int dz D_{c\to D}(z) + \int dz D_{c\to D^*}(z)}\ , \tag{11}$$

which is a function of r only. The prediction by the PQCD fragmentation model is shown by the solid curve in Fig. 5. At $r=0$, the heavy quark mass limit, $P_V = 0.75$, which is

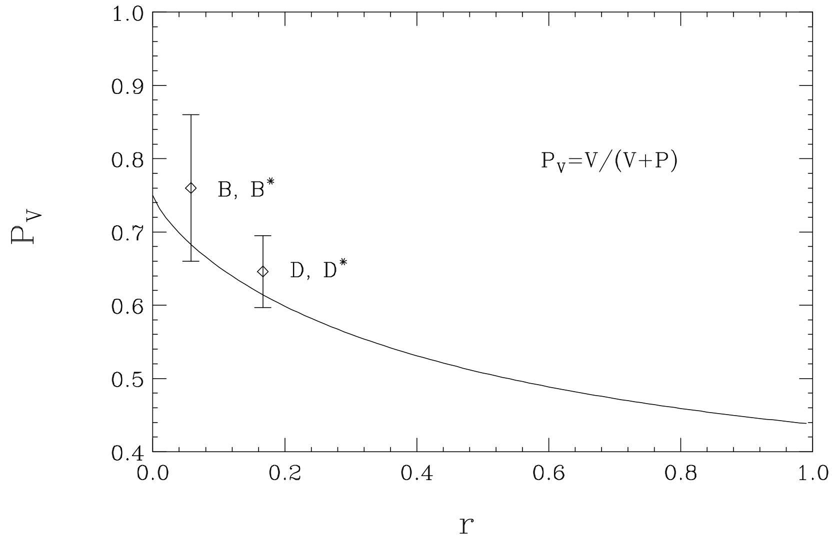

Figure 5: The ratio $P_V = V/(V+P)$, where V denotes the vector state (D^*, B^*) and P denotes the pseudoscalar (D, B). The data for D, D^* system and for B, B^* system are shown.

exactly the value given by the naive spin counting. At $r > 0$ P_V is always smaller than 0.75, which implies that D^* is produced less than given by heavy quark spin symmetry. This can be understood in terms of the mass splitting between D and D^* mesons, which can be accounted for by the $\sigma^{\mu\nu}G_{\mu\nu}/M$ term in the HQET.

A compilation of data on P_V can be found in Ref. [31], in which the updated branching ratio $B(D^{*+} \to D^0\pi^+) = 0.681 \pm 0.016$ was used, and the average $P_V = 0.646 \pm 0.049$. This value of P_V also indicates that D^* mesons are produced less than it should be as given by heavy quark spin symmetry. For the charm quark we take $m_c = 1.5$ GeV and the light constituent quark mass to be 0.3 GeV, therefore $r = \frac{m_{\rm light}}{m_c + m_{\rm light}} = 0.167$. The data is then plotted on the graph and very good agreement is obtained. Recently, the data for B, B^* system has also been available with $P_V(B, B^*) = 0.76 \pm 0.08 \pm 0.06$ [32]. We take $m_b = 4.9$ GeV and $m_{\rm light} = 0.3$ GeV again, therefore $r = 0.06$. The prediction is still less than 1σ from the data point. From the figure we can see that if we choose a smaller value of $m_{\rm light}$, say 0.2 GeV, we could even get a better agreement. The errors in P_V certainly allow us to vary $m_{\rm light}$ more than 0.15 GeV such that the prediction is still within 1σ. Or, we can use the experimental value of P_V to fix the parameter r.

B) $\langle z\rangle$ is the average longitudinal momentum fraction that is transferred from the heavy quark to the meson. In terms of fragmentation functions, $\langle z\rangle^\mu$ at a scale μ is given by

$$\langle z\rangle^\mu_{c\to D^*} = \frac{\int dz\, z D_{c\to D^*}(z,\mu)}{\int dz\, D_{c\to D^*}(z,\mu)} \ . \tag{12}$$

Experimentally, the inclusive $c \to D^*$ channel was measured at LEP, at CLEO, and at ARGUS. The $\langle z\rangle^\mu_{c\to D^*}$ given in Eqn. (12) is the ratio of the second to the first moments of the fragmentation function at the scale μ. Since the anomalous dimensions of the moments

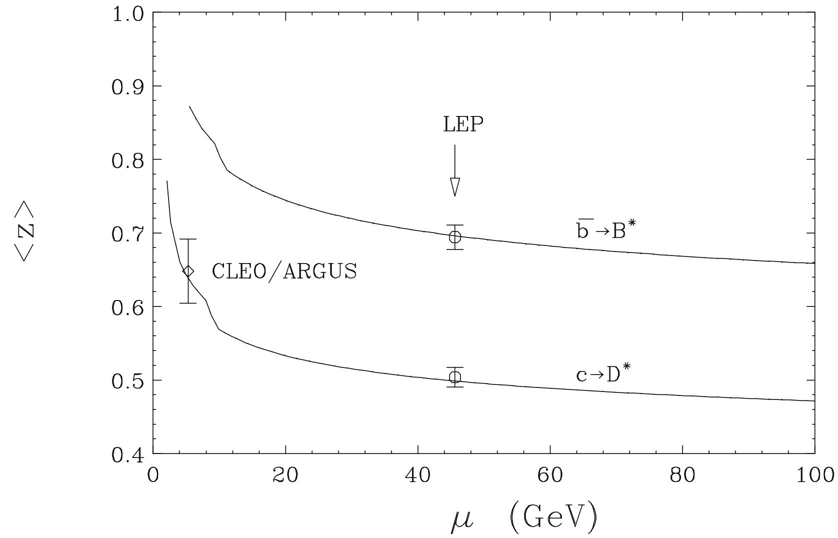

Figure 6: The average $\langle z\rangle^\mu$ for $c \to D^*$ and for $\bar{b} \to B^*$ fragmentation versus the scale μ. The experimental measurements from LEP ($\mu = m_Z/2$) and from CLEO/ARGUS ($\mu = 5.3$ GeV) are shown.

are known explicitly, the scaling behavior of $\langle z\rangle^\mu$ can be determined to be

$$\langle z\rangle^\mu = \langle z\rangle^{\mu_0} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{-2\gamma}{b}} \quad , \tag{13}$$

where $\gamma = -4C_F/3$, $C_F = 4/3$, $b = (11N_c - 2n_f)/3$, $N_c = 3$, n_f is the number of active flavors at the scale μ, and $\langle z\rangle^{\mu_0}$ is the value determined at the initial scale μ_0. Taking the inputs: $m_c = 1.5$ GeV, $m_{u,d} = 0.3$ GeV, $\mu_0 = m_c + 2m_{u,d} = 2.1$ GeV, we have $r = 0.167$, and $\langle z\rangle^{\mu_0}_{c\to D^*} = 0.77$ and $\langle z\rangle^{\mu=m_Z/2}_{c\to D^*} = 0.50$. The variation of $\langle z\rangle^{\mu}_{c\to D^*}$ and $\langle z\rangle^{\mu}_{\bar{b}\to B^*}$ as functions of μ are shown in Fig. 6, where we chose $m_b = 4.9$ GeV. The curve for $\langle z\rangle^{\mu}_{\bar{b}\to B^*}$ was also shown because we want to demonstrate that using the same $m_{u,d}$ and $m_b = 4.9$ GeV, the results predicted also agree with the data for the bottom quark fragmentation at LEP.

The measured quantity is $\langle x_E\rangle$, which is the energy of the meson relative to one half of the center-of-mass energy of the machine, and x_E should be a good approximation to z. At LEP, the average value of $\langle x_E\rangle_{c\to D^*} = 0.504 \pm 0.0133$ [33]. For the bottom quark, only the inclusive hadron production has been measured. But we expect that $\langle x_E\rangle_{b\to B^*}$ should be close to $\langle x_E\rangle_{b\to H_b}$, where H_b is a bottom hadron, because the $b \to B^*$ is the dominant fragmentation mode of the bottom quark. The average value of $\langle x_E\rangle_{b\to H_b} = 0.694 \pm 0.0166$ [34]. Also, we have data on $c \to D^*$ from CLEO and ARGUS [35]. Combining the CLEO and ARGUS data we have $\langle x_E\rangle_{c\to D^*} = 0.648 \pm 0.043$. The scale of the measurements is taken to be one half of the center-of-mass energy of the machines, so it is $m_Z/2$ at LEP and 5.3 GeV at CLEO/ARGUS. These data are shown in Fig. 6. Excellent agreement is demonstrated. The only inputs to these comparisons are simply r and μ_0. Once they are fixed, $\langle z\rangle^{\mu_0}$ can be calculated by Eqn. (12) and evolved by Eqn. (13) to any scale μ. The results show agreement at two different scales. This is a big contrast to Peterson fragmentation model, which fits to different values of mass ratio ϵ_Q at different scales.

V. Conclusions

In this proceedings I have summarized some recent work on PQCD fragmentation functions. The ψ production at the Tevatron is the first evidence showing the importance of parton fragmentation in quarkonium production in the large p_T region. I have also shown the results of the production of $\bar{b}c$ mesons by the fragmentation approach. Finally, a fragmentation model based on the PQCD fragmentation functions is advocated to describe the fragmentation of heavy quarks into heavy-light mesons. This model lies on a firm basis of PQCD and is consistent with HQET, and was successfully applied to fit data on P_V and $\langle z \rangle^\mu$. Other work includes the estimation of the strange-quark mass parameter m_s in the B_s and B_s^* system, in which the probability of a b quark going into a stranged B meson is fitted to the value of m_s [36], and a value of about 300 MeV was obtained. There was also a calculation [37] of the Falk-Peskin variable $w_{3/2}$ [31] using the PQCD fragmentation functions. Another work was the extension to the fragmentation of a heavy quark into a baryon containing two heavy quarks [38].

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